

March 1906. *Mr. Cowell, Greenwich Observations of Venus.* 307

re-reduced with the same care that has lately been bestowed on several of the earlier star catalogues. I consider therefore that Professor Newcomb has given excessive weight to his other material, and that it would have been better to leave Le Verrier's mass of *Venus* unaltered.

Reciprocal of Mass of *Venus*.

Le Verrier ...	...	...	...	400×10^3
Newcomb ...	...	...	...	408×10^3
Present paper	...	...	...	399×10^3

Discussion of Greenwich Observations of Venus, 1869-1900.
By P. H. Cowell, M.A.

The observations under examination are divided into four periods of analysis of eight years each. Eight years is equivalent to thirteen revolutions of *Venus*, or five synodic periods. For each period of analysis corrections to the mean longitude, eccentricity, and perihelion of both the Earth and *Venus* are obtained. A correction of about 3" to Le Verrier's centennial motion of *Venus* is distinctly indicated, but a period of thirty-two years is naturally too short to obtain reliable corrections proportional to the time. I have therefore merely taken means for the four periods of analysis, and thus obtained corrections applicable to Le Verrier's elements for 1885.0.

The peculiarity of my analysis is that it contains no simultaneous equations, a feature which I believe will be appreciated by readers of Le Verrier, vol. vi. pp. 58-70, or Newcomb, *Astronomical Constants*, p. 92. This has been achieved by forcing symmetry into the distribution of observations by giving enhanced weight, whenever necessary, to isolated observations. The process no doubt slightly diminishes the weight of the results as compared with an ideal method, but, as I shall now show, it involves a considerable increase of accuracy as compared with the more laborious methods just referred to. In Newcomb's *Astronomical Constants*, p. 57, we read: "In dealing with so extensive a system of unknown quantities it is impracticable to investigate the dependence of each upon all the others." Now it is just this dependence, which Newcomb found it impracticable to investigate, which is brought out in the present paper. And the investigation is by no means unnecessary; for I shall show that the corrections to the eccentricity and perihelion of the Earth are determined by a vector quantity whose components are $y+z$, $y'+z'$. A similar statement holds for *Venus* with components $y+2z$, $y'+2z'$. It also appears that y , y' are quantities that can only be determined with 10 per cent. of the weight

of z, z' . Now when the dependence of each of the unknown quantities upon the others is not investigated, the errors of *Venus* are determined with no greater weight than y, y' . In the present analysis it is practicable to adopt the value of the vector $y+z, y'+z'$ directly from the observations of the Sun, and to pass to *Venus* by adding the well-determined vector z, z' , thus obtaining $y+2z, y'+2z'$. Owing to the enormous number of observations available, there is no great difference between the numerical results of the two processes.

It is necessary to exhibit results by resolving into corrections to eccentricity and perihelion. This can be done, however, after the analysis; for the purposes of analysis it is preferable to resolve towards and at right angles to the direction of conjunction. During any period of analysis therefore, V, E will denote the mean longitudes of *Venus* and Earth measured from the common longitude of the third (or middle of five) inferior conjunction.

Period of Analysis.	Longitude of Third Inferior Conjunction.	Date.
1869-1876	$222^{\circ}5$	1873 May 4
1877-1884	$220^{\circ}2$	1881 May 1
1885-1892	$217^{\circ}9$	1889 April 29
1893-1900	$215^{\circ}6$	1897 April 26

The variables x, x', y, y', z, z' are defined thus: the corrections to the longitudes of *Venus* and Earth are

$$\begin{aligned}\delta V &= x + x' + 2(y + 2z) \sin V + 2(y' + 2z') \cos V \\ \delta E &= x' + 2a(y + z) \sin E + 2a(y' + z') \cos E\end{aligned}$$

then, in order to conform to Kepler's laws, it is necessary to put for the variations of the radii vectores

$$\begin{aligned}\delta r' &= -a(y + 2z) \cos V + a(y' + 2z') \sin V \\ \delta r &= -a(y + z) \cos E + a(y' + z') \sin E\end{aligned}$$

where a = ratio of mean distances = 0.7233 .

It will be noticed that if y, y' alone are different from zero the displacements of the two planets are equal and parallel at all conjunctions; the geocentric displacement of *Venus* is therefore zero; y, y' therefore must be determined from observations remote from conjunction; the small weights with which they are determined may therefore be anticipated.

In forming the equations of condition the utmost simplicity is desirable. Complex equations are laborious, both to form and to solve. At the worst, the modifications introduced for the sake of simplicity will only cause the solution obtained to be an approximation to, instead of identical with, the solution of accurate equations. The approximate solution could be carried accurately into the tabular places, and fresh residuals formed,

Period I. 1869-1876. Unit 0^s.01.
 Longitude at middle inferior conjunction = 222°·5.
 Date 1873 May 4.

Mean Error of Right Ascension of Venus.

V-E.	0.	1.	2.	3.	4.	5.	Sum.
180°	...	+ 19	(+ 12)	+ 2	(+ 8)	(+ 6)	+ 47
195	...	+ 2	+ 9	+ 9	+ 7	- 3	+ 24
210	...	+ 13	(+ 11)	+ 6	+ 10	+ 3	+ 43
225	...	+ 29	+ 13	+ 14	+ 16	(+ 5)	+ 77
240	...	- 8	+ 9	+ 22	+ 16	(+ 6)	+ 45
255	...	+ 7	+ 21	(+ 15)	+ 7	+ 10	+ 60
270	...	+ 10	+ 2	+ 13	+ 12	+ 9	+ 46
285	...	+ 5	+ 19	+ 15	+ 16	+ 14	+ 69
300	...	+ 1	(+ 10)	+ 8	+ 5	+ 18	+ 42
315	...	- 4	+ 10	+ 6	+ 22	+ 23	+ 57
330	...	0	+ 10	+ 8	+ 11	+ 22	+ 51
345	...	- 7	+ 8	+ 8	+ 8	+ 15	+ 32
0	...	- 13	- 18	- 2	- 6	- 6	- 45
15	...	- 10	- 9	- 4	(- 6)	+ 5	- 24
30	...	- 4	- 6	+ 5	- 8	- 4	- 17
45	...	- 2	- 1	- 3	+ 2	+ 10	+ 6
60	...	0	+ 2	+ 3	+ 4	+ 7	+ 16
75	...	+ 4	+ 3	+ 4	+ 4	+ 9	+ 24
90	...	+ 5	- 1	+ 10	+ 12	+ 3	+ 29
105	- 9	+ 12	- 5	+ 8	+ 1	...	+ 7
120	- 8	+ 7	+ 1	+ 3	+ 15	...	+ 18
135	+ 2	+ 7	+ 6	+ 15	+ 5	...	+ 35
150	(+ 1)	+ 14	+ 9	0	+ 8	...	+ 32
165	0	+ 14	+ 16	+ 10	+ 10	...	+ 50
General Sum							+ 724

and a second approximation obtained with the simple equations of condition. The simplifications that I am about to introduce, however, do not appear to prevent the solutions from being accurate to within 5 or 10 per cent., and the imperfections of the observations themselves probably preclude a higher degree of accuracy. The errors of Le Verrier's tables are in fact so small that 5 per cent. of their amount is too small a quantity to determine. The simplifications are as follows :

In forming the variation of geocentric longitude in terms of the errors of heliocentric coordinates, the orbits are supposed circular and co-planar, the mean motions during one period of analysis are taken as accurately in the ratio 13 : 8, and an error

Period II. 1877-1884. Unit 0^s.01.
Longitude at middle inferior conjunction = 220° 2.
Date 1881 May 1.

Mean Error of Right Ascension of Venus.							
V-E.	0.	1.	2.	3.	4.	5.	Sum.
180°	...	(+ 2)	(0)	(+10)	(0)	(+ 6)	+ 18
195	...	+ 6	0	(+10)	- 1	(+ 6)	+ 21
210	...	+17	(+ 3)	+ 5	+10	+ 6	+ 41
225	...	+ 6	+ 7	+18	+22	+12	+ 65
240	...	+ 2	+ 8	+ 2	+20	(+ 6)	+ 38
255	...	+ 7	+16	+15	+ 8	+ 2	+ 48
270	...	+11	+14	+ 8	+ 4	+ 5	+ 42
285	...	+ 2	+ 6	+ 5	+ 5	+17	+ 35
300	...	+ 6	+ 4	+ 8	+12	+ 6	+ 36
315	...	- 1	+17	(+ 9)	- 1	+11	+ 35
330	...	+ 5	+10	+11	+ 5	+ 4	+ 35
345	...	- 8	+13	+ 3	-29	+13	- 8
0	...	- 2	+ 3	- 6	-55	+ 3	- 57
15	...	- 6	- 5	- 4	-34	+ 3	- 46
30	...	- 5	+ 1	+ 6	- 7	+ 6	+ 1
45	...	- 2	+ 2	+11	- 4	+ 7	+ 14
60	...	+ 8	+ 6	+13	+ 3	+ 5	+ 35
75	...	+ 7	+22	+ 7	+ 7	+15	+ 58
90	...	+ 1	+ 1	+ 9	+ 6	+ 9	+ 26
105	(+ 1)	+ 7	- 6	+ 6	+ 5	...	+ 13
120	0	+ 2	+12	+ 1	+ 8	...	+ 23
135	-18	+ 2	(+10)	0	+13	...	+ 7
150	- 2	+ 5	+ 8	- 6	+ 5	...	+ 10
165	(0)	0	+15	(0)	+ 7	...	+ 22
General Sum							+512

of right ascension is treated as equivalent to an error of longitude.

We therefore put $V = 13\theta$ $E = 8\theta$ $V - E = \phi = 5\theta$.
For the geocentric distance Δ we have $\Delta^2 = 1 - 2a \cos \phi + a^2$.
For the variation of geocentric longitude

$$d\lambda = \frac{a}{\Delta^2} \left\{ \left(-\frac{dr'}{a} + dr \right) \sin (V - E) + \left(a dV + \frac{dE}{a} \right) - (dV + dE) \cos (V - E) \right\}$$

whence substituting the values of dr' &c. in terms of $x, x'...$

Period III. 1885-1892. Unit 0^s.01.
Longitude at middle inferior conjunction = 217°9.
Date 1899 April 29.

Mean Error of Right Ascension of Venus.

V-E.	0.	1.	2.	3.	4.	5.	Sum.
180°	...	+ 10	(+ 6)	(+ 10)	(+ 6)	(+ 7)	+ 39
195	...	+ 12	(+ 8)	+ 8	+ 6	+ 3	+ 37
210	...	+ 14	+ 10	- 10	+ 5	+ 21	+ 40
225	...	+ 11	+ 14	+ 7	+ 12	+ 9	+ 53
240	...	+ 14	+ 12	+ 21	+ 11	+ 13	+ 71
255	...	+ 14	+ 17	+ 8	+ 5	+ 18	+ 62
270	...	+ 8	+ 12	+ 11	+ 5	+ 8	+ 44
285	...	+ 16	+ 9	+ 7	+ 4	+ 4	+ 40
300	...	+ 3	+ 11	+ 5	- 3	+ 10	+ 26
315	...	+ 4	+ 21	+ 2	+ 7	+ 8	+ 42
330	...	+ 4	+ 8	+ 15	- 4	+ 10	+ 33
345	...	0	- 9	+ 8	- 23	+ 15	- 9
0	...	- 6	- 30	- 11	- 55	+ 2	- 100
15	...	- 7	- 23	+ 1	- 20	+ 15	- 34
30	...	- 1	- 10	+ 10	- 6	+ 16	+ 9
45	...	- 1	0	+ 6	+ 2	+ 8	+ 15
60	...	+ 4	0	+ 9	+ 9	+ 20	+ 42
75	...	+ 8	+ 11	+ 14	- 9	(+ 15)	+ 39
90	...	+ 19	+ 9	+ 17	+ 4	+ 14	+ 63
105	(+ 6)	+ 14	+ 14	+ 8	+ 10	...	+ 52
120	(+ 3)	+ 4	+ 5	+ 12	+ 12	...	+ 36
135	+ 1	+ 8	- 10	+ 11	+ 8	...	+ 18
150	- 3	+ 3	+ 14	+ 6	+ 10	...	+ 30
165	- 3	+ 5	+ 11	+ 8	(+ 8)	...	+ 29
General Sum							+ 677

$d\lambda = ax + a'x + by + b'y' + cz + c'z' = \epsilon = \text{tab.}-\text{obs. error of R.A.}$

where $a = \frac{a}{\Delta^2} \{a - \cos 5\theta\}$

$a' = 1$

$b = \frac{a}{\Delta^2} \sin 5\theta \{-\cos 13\theta + a \cos 8\theta\}$

$b' = \frac{a}{\Delta^2} \sin 5\theta \{\sin 13\theta - a \sin 8\theta\}$

B B

Period IV. 1893-1900. Unit 0^s.01.
 Longitude at middle inferior conjunction = 215°6.
 Date 1897 April 26.

Mean Error of Right Ascension of Venus.

V-E.	0.	1.	2.	3.	4.	5.	Sum.
180°	...	(+13)	(+13)	(+15)	+18	(+15)	+74
195	...	+16	(+16)	+10	+9	+17	+68
210	...	+16	+19	+9	+7	+16	+67
225	...	+15	+12	+4	+16	+12	+59
240	...	+4	+14	+10	+25	+25	+78
255	...	+13	+15	+12	+7	+9	+56
270	...	+11	+17	+12	+2	+14	+56
285	...	+10	+13	+8	+6	+10	+47
300	...	+10	+11	(+7)	+10	+7	+45
315	...	+8	+12	+6	+11	+11	+48
330	...	-2	+11	+8	-10	+12	+19
345	...	-12	+6	+6	-16	+16	0
0	...	-23	-14	0	35	+18	-54
15	...	-14	0	+1	-20	+13	-20
30	...	-6	-1	+16	-4	+15	+20
45	...	-6	+13	+19	+5	+14	+45
60	...	+7	+18	+21	+15	+17	+78
75	...	+20	+2	+19	+5	+20	+66
90	...	+21	+12	+12	+8	+11	+64
105	...	+20	+5	+13	+11	+7	+56
120	(+10)	+11	+6	+8	+9	...	+44
135	+5	+10	+12	+12	+17	...	+56
150	+2	+13	+10	+14	+11	...	+50
165	+11	+11	+15	-11	+13	...	+39
General Sum						...	1061

$$c = \frac{a}{\Delta^2} [-\sin 18\theta + \frac{5}{2}a \sin 13\theta - \sin 8\theta - \frac{1}{2}a \sin 3\theta]$$

$$c' = \frac{a}{\Delta^2} [-\cos 18\theta + \frac{5}{2}a \cos 13\theta - \cos 8\theta - \frac{1}{2}a \cos 3\theta]$$

The mean values of aa' , ab , ab' are all zero. Therefore, when symmetry has been imported into the observations by suitably manipulating the weights, the normal equations are

$$x = \Sigma a\epsilon \div \Sigma a^2 \text{ with a weight } \Sigma a^2 \text{ \&c.}$$

We now form tables from the preceding formulæ.

TABLE I.

<i>V-E.</i>	<i>a.</i>	<i>Value of a. Unit = 0.01. Mean Square = 0.55.</i>
180	+ 42	
195	+ 42	
210	+ 41	
225	+ 41	
240	+ 39	
255	+ 37	
270	+ 34	
285	+ 29	
300	+ 20	
315	+ 2	
330	- 38	
345	- 140	
0	- 261	
15	- 140	
30	- 38	
45	+ 2	
60	+ 20	
75	+ 29	
90	+ 34	
105	+ 37	
120	+ 39	
135	+ 41	
150	+ 41	
165	+ 42	

The quantities in Tables I.-VI. are set down to two places of decimals, but they are subsequently used to one place only.

TABLE II.

$a' = 1.0.$ *Mean Square = 1.0.*

$$\frac{1-a^2}{\Delta^2} = 1 + 2 \sum_{j=1}^{\infty} a^j \cos j\phi$$

$$\frac{(1-a^2)^3}{\Delta^4} = 1 + a^2 + 2 \sum_{j=1}^{\infty} a^j \cos j\phi [(j+1) - (j-1)a^2]$$

whence the following mean values may be obtained of

$$\frac{1-a^2}{\Delta^2} = 1$$

$$\frac{(1-a^2) \cos 2\phi}{\Delta^2} = a^2$$

$$\frac{(1-a^2) \cos \phi}{\Delta^2} = a$$

$$\frac{(1-a^2) \cos^2 \phi}{\Delta^2} = \frac{1}{2}(1+a^2)$$

B B 2

TABLE III.
Value of *b*. Unit 0.01. Mean Square = 0.13.

V-E.	1.	2.	3.	4.	5.
180	0	0	0	0	0
195	- 9	+ 4	+ 3	- 8	+ 10
210	- 9	- 4	+ 16	- 22	+ 19
225	+ 5	- 22	+ 31	- 29	+ 15
240	+ 28	- 41	+ 39	- 21	- 4
255	+ 49	- 48	+ 28	+ 2	- 32
270	+ 55	- 34	0	+ 34	- 55
285	+ 40	- 2	- 37	+ 62	- 63
300	+ 3	+ 39	- 65	+ 67	- 43
315	- 41	+ 68	- 69	+ 44	- 2
330	- 68	+ 65	- 37	- 5	+ 45
345	- 42	+ 15	+ 17	- 44	+ 53
0	0	0	0	0	0
15	- 53	+ 44	- 17	- 15	+ 42
30	- 45	+ 5	+ 37	- 65	+ 68
45	+ 2	- 44	+ 69	- 68	+ 41
60	+ 43	- 67	+ 65	- 39	- 3
75	+ 63	- 62	+ 37	+ 2	- 40
90	+ 55	- 34	0	+ 34	- 55
105	+ 32	- 2	- 28	+ 48	- 49
120	+ 4	+ 21	- 39	+ 41	- 28
135	- 15	+ 29	- 31	+ 22	- 5
150	- 19	+ 22	- 16	+ 4	+ 9
165	- 10	+ 8	- 3	- 4	+ 9

$$\frac{\sin^2 \phi}{\Delta^2} = \frac{1}{2}$$
$$\frac{(1-a^2)^3}{\Delta^4} = 1 + a^2$$
$$\frac{(1-a^2)^3 \cos \phi}{\Delta^4} = 2a$$
$$\frac{(1-a^2)^3 \cos 2\phi}{\Delta^4} = a^2(3-a^2)$$
$$\frac{(1-a^2)^3 \cos 3\phi}{\Delta^4} = a^3(4-2a^2)$$

$$\frac{(1-a^2)^3 \cos^2 \phi}{\Delta^4} = \frac{1}{2}(1+4a^2-a^4)$$
$$\frac{(1-a^2) \sin^2 \phi}{\Delta^4} = \frac{1}{2}$$
$$\frac{(1-a^2)^3 \cos 3\phi}{\Delta^4} = \frac{a}{2}(3+2a^2-a^4)$$
$$\frac{(1-a^2) \cos \phi \sin^2 \phi}{\Delta^4} = \frac{a}{2}$$

Considerations of symmetry will show that the mean values of twelve out of the fifteen quantities *aa'* &c. are zero.
The above formulæ will readily show that *aa'*, *bc*, *b'c'* also take zero mean values.

TABLE IV.
Value of b' . Unit 0.01. Mean Square = 0.13.

V-E.	1.	2.	3.	4.	5.
180	0	0	0	0	0
195	+ 6	- 10	+ 10	- 6	+ 1
210	+ 20	- 21	+ 15	- 2	- 11
225	+ 32	- 23	+ 5	+ 14	- 28
240	+ 32	- 9	- 17	+ 36	- 42
255	+ 14	+ 18	- 43	+ 51	- 40
270	- 18	+ 47	- 58	+ 47	- 18
285	- 52	+ 65	- 54	+ 22	+ 18
300	- 70	+ 58	- 24	- 19	+ 55
315	- 59	+ 24	+ 21	- 57	+ 72
330	- 17	- 26	+ 59	- 70	+ 53
345	+ 32	- 51	+ 50	- 30	- 1
0	0	0	0	0	0
15	- 1	- 30	+ 50	- 51	+ 32
30	+ 53	- 70	+ 59	- 26	- 17
45	+ 72	- 57	+ 21	+ 24	- 59
60	+ 55	- 19	- 24	+ 58	- 70
75	+ 18	+ 22	- 54	+ 65	- 52
90	- 18	+ 47	- 58	+ 47	- 18
105	- 40	+ 51	- 43	+ 18	+ 14
120	- 42	+ 36	- 17	- 9	+ 32
135	- 28	+ 14	+ 5	- 23	+ 32
150	- 11	- 2	+ 15	- 21	+ 20
165	+ 1	- 6	+ 10	- 10	+ 6

We also obtain

mean value of $a^2 = \frac{1}{2} \frac{a^2}{1-a^2}$

" " $a'^2 = 1$

" " $b^2 \text{ or } b'^2 = \frac{1}{4}a^2$

" " $c^2 \text{ or } c'^2 = \frac{a^2(1+\frac{1}{4}a^2)}{1-a^2}$

The observations are divided into groups of 24.33006 days each ; the centre of each group corresponds to an exact multiple of 15° in the mean heliocentric elongation. There are 120 groups in each period of analysis. The mean error of right ascension is given in the following tables. Whenever an error is enclosed in brackets, it implies there were no observations in the corresponding group, but an error has been interpolated from

TABLE V.

Value of c . Unit 0.01. Mean Square = 1.24.

V—E.	1.	2.	3.	4.	0 or 5.
180	0	+ 49	— 80	+ 80	— 49
195	— 57	+ 83	— 78	+ 42	+ 9
210	— 87	+ 77	— 37	— 17	+ 65
225	— 77	+ 32	+ 24	— 72	+ 92
240	— 30	— 30	+ 78	— 97	+ 78
255	+ 33	— 82	+ 101	— 81	+ 30
270	+ 83	— 103	+ 83	— 32	— 32
285	+ 101	— 86	+ 37	+ 26	— 79
300	+ 86	— 46	— 11	+ 64	— 92
315	+ 60	— 21	— 27	+ 64	— 77
330	+ 86	— 59	+ 10	+ 43	— 80
345	+ 271	— 216	+ 78	+ 90	— 224
0	+ 497	— 308	0	+ 308	— 497
15	+ 224	— 90	— 78	+ 216	— 271
30	+ 80	— 43	— 10	+ 59	— 86
45	+ 77	— 64	+ 27	+ 21	— 60
60	+ 92	— 64	+ 11	+ 46	— 86
75	+ 79	— 26	— 37	+ 86	— 101
90	+ 32	+ 32	— 83	+ 103	— 83
105	— 30	+ 81	— 101	+ 82	— 33
120	— 78	+ 97	— 78	+ 30	+ 30
135	— 92	+ 72	— 24	— 32	+ 77
150	— 65	+ 17	+ 37	— 77	+ 87
165	— 9	— 42	+ 78	— 83	+ 57

neighbouring observations. It will be seen that bracketed errors are most frequent at superior conjunction, but seldom occur elsewhere.

The analysis of the errors consists simply in multiplying each entry by the corresponding entry of Tables I.—VI. and dividing by 120 times the mean square of the factor.

For example, Period I. 1869–1876, to find x .

The formula is $x = \Sigma a\epsilon \div \Sigma a^2$, where ϵ denotes an entry in the table of Period I., and the values of a are given in Table I. As the value of a is a function of V—E only, we naturally proceed to add horizontally (the sums are exhibited in the extreme right-hand column).

Similarly $x' = \Sigma a'\epsilon \div \Sigma a'^2 = \frac{\Sigma \epsilon}{120}$, since $a' = 1$. The value

of $\Sigma \epsilon$ is exhibited in the right-hand bottom corner.

TABLE VI.
Value of c' . Unit 0.01. Mean Square = 1.24.

V-E.	1.	2.	3.	4.	0 or 5.
180	- 84	+ 68	- 26	- 26	+ 68
195	- 63	+ 18	+ 34	- 74	+ 85
210	- 10	- 43	+ 80	- 86	+ 59
225	+ 51	- 86	+ 89	- 57	+ 4
240	+ 92	- 92	+ 57	0	- 57
255	+ 95	- 58	- 2	+ 61	- 96
270	+ 61	0	- 61	+ 98	- 98
285	+ 6	+ 55	- 94	+ 98	- 64
300	- 39	+ 82	- 93	+ 69	- 19
315	- 48	+ 74	- 72	+ 43	+ 3
330	- 18	+ 65	- 88	+ 77	- 37
345	- 6	+ 166	- 261	+ 256	- 153
0	- 161	+ 423	- 523	+ 423	- 161
15	- 153	+ 256	- 261	+ 166	- 6
30	- 37	+ 77	- 88	+ 65	- 18
45	+ 3	+ 43	- 72	+ 74	- 48
60	- 19	+ 69	- 93	+ 82	- 39
75	- 64	+ 98	- 94	+ 55	+ 6
90	- 98	+ 98	- 61	0	+ 61
105	- 96	+ 61	- 2	- 58	+ 95
120	- 57	0	+ 57	- 92	+ 92
135	+ 4	- 57	+ 89	- 86	+ 51
150	+ 59	- 86	+ 80	- 43	- 10
165	+ 85	- 74	+ 34	+ 18	- 63

The details of the calculation of y, y', z, z' are not exhibited : they consist merely in 120 multiplications. The factors used are given in Tables III., IV., V., and VI., but one less significant figure is employed.

Multiplying by 15, to transform to seconds of arc, we thus get the following table of results :

Period.	Long. of Con- junction.	Date.	Wt. 55. x	Wt. 100. x'	Wt. 13. y	Wt. 13. y'	Wt. 124. z	Wt. 124. z'
I.	222.5	1873 May 4	+ 0.75	+ 0.90	+ 0.44	0.00	- 0.23	- 0.15
II.	220.2	1881 1	+ 0.90	+ 0.64	+ 0.55	+ 0.18	- 0.57	- 0.41
III.	217.9	1889 Apr. 29	+ 1.27	+ 0.84	+ 0.54	+ 0.15	- 0.33	- 0.61
IV.	215.6	1897 26	+ 1.15	+ 1.32	+ 0.84	- 0.22	- 0.60	- 0.42
Means	219 0		+ 1.02	+ 0.92	+ 0.59	+ 0.03	- 0.43	- 0.40

This completes the analysis : it is impossible to get reliable terms proportional to the time from thirty-two years' observations, but the run in the values of $x+x'$, the correction to the mean longitude of *Venus*, approximately confirms Newcomb's diminution by $3''$ of Le Verrier's centennial motion.

The mean results apply to the date 1885.0.

From the Sun observations I obtained as a correction to the Sun's longitude

$$\begin{aligned} (\text{tab.} - \text{obs.}) & + 0''.78 - 0''.47 \sin (E - 115^\circ) \\ \text{or} & + 0''.78 - 0''.46 \cos (E - 219^\circ) + 0''.11 \sin (E - 219^\circ) \end{aligned}$$

Dividing the last two coefficients by $2a = 1.447$, we have from the Sun observations and in the notation of this paper

$$x' = +0''.78 \quad y+z = +0''.08 \quad y'+z' = -0''.31 \quad \dots \quad (\text{i.})$$

These values are entitled to far greater weight than the corrections to the solar elements obtained from *Venus*

$$x' = +0''.92 \quad y+z = +0''.16 \quad y'+z' = -0''.37 \quad \dots \quad (\text{ii.})$$

It will be seen, however, that the accordance is most satisfactory.

In passing to the corrections for *Venus* I use values (i.) and reject values (ii.) for x' , $y+z$, $y'+z'$. This makes it possible to get corrections for *Venus* independently of the values of y , y' .

We thus have for *Venus*

$$x+x' = +1''.80 \quad y+2z = -0''.35 \quad y'+2z' = -0''.71$$

or Le Verrier's longitude of *Venus* requires corrections in 1885.0,

$$\begin{aligned} & -1''.80 + 2 \times 0''.35 \sin (V - 219^\circ) + 2 \times 0''.71 \cos (V - 219^\circ) \\ & = -1''.80 + 2 \times 0''.72 \sin (V - 130^\circ) - 2 \times 0''.34 \cos (V - 130^\circ), \end{aligned}$$

on resolving in the direction of the perihelion of *Venus*.

Hence corrections required by Le Verrier's elements in 1885.0 are

Element.	Correction required by Le Verrier in 1885.0.		
Mean longitude	...	...	$-1''.80$
Eccentricity	...	...	$+0''.72$
Coefficient of principal elliptic term in longitude	...	...	$+1''.44$
Mean anomaly	...	...	$-0''.34 \div e = -50''$
Perihelion	...	...	$+48''$

It is interesting to compare these corrections with the differences between Le Verrier's tables and Newcomb's.

Counting T in centuries from 1900.0 we have the following differences, Newcomb-Le Verrier :

Formula.

Element.	For Difference Newcomb-Le Verrier.		Value in 1885.0.
Mean longitude ...	$- 1''.74 -$	$3''.29T - 0''.0141T^2$	$- 1''.25$
Perihelion ...	$+ 83''.9$	$+ 128''.66T + 2''.415T^2$	$+ 64''.6$
Principal elliptic co- efficient in longi- tude ...	$+ 1''.79 +$	$2''.518T - 0''.015T^2$	$+ 1''.41$

There is thus most satisfactory agreement between the eccentricity found in this paper and that used by Newcomb. The mean longitude and perihelion cannot be compared unless allowance is made

(i.) For any changes of long-period terms between Le Verrier's tables and Newcomb's.

(ii.) For the difference $0''.8$ between Newcomb's equinox and that used in the Greenwich clock-star list up to and including 1902.

It will be seen that (ii.) approximately explains the difference between Newcomb's correction to the mean longitude and that obtained in this paper.

It will be seen that the analyses of this paper are unaffected by the following alterations which involve odd functions of the elongation only in the geocentric places :

(i.) A change in Le Verrier's mass of the Earth + Moon which will introduce corrections varying as $\sin(V-E)$, $\sin 2(V-E)$, &c. with the tabular geocentric places.

(ii.) A change in the tabular semidiameter and Stone's empirical alteration thereof, which, as it takes opposite signs before and after conjunction, can presumably be expanded in a form

$$\pm \mu + a_1 \sin(V-E) + a_2 \sin 2(V-E) + \dots$$

I have not yet examined how far it is possible to disentangle errors of defective illumination from errors due to Le Verrier's mass of the Earth + Moon, and so obtain a determination of this mass ; but it is fairly obvious that no such determination can compete with the value resulting from pendulum experiments.

It is sufficient for the purposes of this paper to point out that the mean value of the product of either of the above two corrections by any of the six quantities a, a', b, b', c, c' is by symmetry zero.

Comparison of Carrington's Circumpolar Catalogue with the Greenwich-Groombridge System. By W. G. Thackeray.

(Communicated by the Astronomer Royal.)

The forthcoming Greenwich Catalogue for 1900 will consist of two parts : Part I., Fundamental and Zodiacal Stars ; Part II., Astrographic Reference Stars. This latter part consists of some 10,000 circumpolar stars within 26° of the pole. As the Carrington Catalogue affords the only data from which the proper motion of the majority of the stars within 9° of the pole can be determined, it is necessary to investigate its systematic errors, and for this purpose a comparison has been made with those stars which are common to it and the Groombridge-Greenwich system. There are ninety-four stars available, which have been grouped in degrees according to the Carrington Catalogue place.

The Carrington Catalogue contains 3735 stars, extending from 0° to 9° polar distance. The positions are based on instrumental errors derived from twenty special polars the places of which were finally determined by reference to double transits of *Polaris* and *Ursæ Minoris* (Introduction, pp. 13, 22-35). Four or more of these stars—two above pole and two below pole whenever possible—were taken for the determination of the meridian error and of the polar point (p. 15). The circle, which was made to turn on its collar and which was divided by Troughton & Simms's well known engine, was not provided with any means for the determination of errors of division. As the circle was not moved by Carrington during the period covered by his catalogue observations the division errors are the same throughout and necessarily remain uncorrected (p. 16). There were four fixed microscopes with which to read the circle, a vertical and horizontal pair, but after the first year only the horizontal pair were used (p. 7). The catalogue stars were observed and arranged in three sub-zones.

The first sub-zone from $45'$ to 4° of N.P.D.

„ second	„	„	4°	„	7°	„
„ third	„	„	7°	„	9°	„

The places of the stars in the first sub-zone depend on practically an equal number of observations above and below pole. The stars in the second sub-zone for 4° to 6° are mostly observed for the first three hours either one above or two below, or two above and one below. For 7° , with few exceptions, and for the third sub-zone, stars are observed entirely above pole.

The results of the comparison taken as corrections to Carrington's Catalogue place given in the following tables are :—